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An integral approach to plasma shape control

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Abstract

A novel representation of the plasma boundary is presented, which employs a B-spline approach to describe the deformation of the plasma surface in an integral way, without resorting to the local properties of the shape as in the gap technique. The evolution of such a curve on the poloidal plane corresponds to that of the plasma shape and a spline controller is designed to counteract disturbances and regulate the plasma shape error during the discharge. The comparison with a gap controller highlights the accuracy in the control capability of the spline controller and its completeness, in the sense that the control of the spline curve implies (but is not implied by) the control of the gaps.

Keywords: Plasma boundary; B-spline; Shape control

1. Introduction

The theory of plasma physics and the experience of large tokamaks have shown that one of the primary requirements to achieve high fusion performances is the capability to create and sustain strongly shaped plasmas.

In this regard, the currently adopted techniques resort to the control of either the flux value or the boundary to first-wall distance evaluated through the reconstruction algorithms at a set of locations opportunely placed on the poloidal cross section of the machine (Hoffmann and Jardin [1], Pironti and Portone [2]).

Being discrete in the representation of the shape, both these descriptions are convenient for the design of the shape controller and in the past few decades have led to robust MIMO designs for some of the most successful devices (Walker et al. [3], Garribba et al. [4]).

Conversely, the gap approach provides a representation of the shape that is obtained through a sampling of the boundary, neglecting its integral behaviour: the same set of gaps may correspond to a wide range of plasma shapes on the poloidal plane. The local nature inherent in this description and some arbitrariness in the choice of the control variables, may yield a certain ambiguity in the interpretation of the shape deforma-

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tion, requiring as a consequence high level decision of some sort, typically given through the experience of the plasma operator.

In this context, we refer to the discrete representations of the boundary obtained via the gap or the flux control point techniques as *pointwise* descriptions, in the sense that their information is relative to specific locations on the poloidal cross section of the machine. On the other hand, a representation that considers the boundary curve as a whole, in this way taking into account the effective curvature of the plasma shape, would be named as *integral*.

Such a representation is actually brought into focus in this paper, where it is addressed by making use of parametric curves derived from a B-spline model approach, similarly to what has been started in Beghi and Cenedese [4,5]. The spline technique, in fact, allows the representation of a wide variety of shapes, preserving inherent properties of regularity, which are apt at the same time to be described through a limited set of characterising parameters, namely the control points.

From a complementary point of view, the boundary of the plasma, given the fluid nature of the ionised gas, shows attributes of regularity in the general smoothness of the surface (with a possible singularity where a magnetic null is present) and is able to freely deform according to the magnetic configuration.

In conclusion, the possibility of reconstructing the boundary of the plasma through B-spline contours appears to be sensible, and the idea of controlling its shape by controlling the motion and the deformation of these curves on the poloidal plane, is consistent with the considerations so far.

2. The boundary description

In order to produce an integral description of the shape, the starting point is the information embedded in the whole flux map of the machine cross section. Considering the boundary flux value known, in fact, it is possible to extract the boundary shape as a two-dimensional curve in $\Re^2$: this curve shows such regularity that is then well suited to be fitted with a planar B-spline with a low number of control points. In doing so, the space of plasma shapes appears to be mappable into the space of spline curves (in Fig. 1, the whole procedure is illustrated).

The parameterisation of the boundary curve by means of the B-splines is implemented following two approaches: a dynamic one, using the active contours technique; a static one, directly fitting the discrete boundary curve extracted from the flux map. As a matter of fact, the active contour approach produces different set of parameters for the same shape, if the shape is the result of different evolutions, while the direct curve fitting of the boundary curve intrinsically holds a *surjective* relationship between the plasma shape space and the parameter space. In fact, this method is chosen because a *quasi-isomorphism* is established between the two spaces. A shape has only one image in the parameter space (*surjectivity*), while there exists a neighbourhood of a plasma shape which maps into the same image in the parameters space (*non-injectivity*).

In principle, the measure of this neighbourhood, the *non-injectivity degree*, could be different for all the shapes and converge to zero for a number of parameters tending to the number of boundary curve samples; it can be reduced, but with the obvious drawback of augmenting the parameter space dimensions. The non-injectivity degree is estimated by analysing a high number of equilibria, exploring in this way the plasma shape space, and then evaluating the distance between the spline curve reconstructed from the parameters and the original boundary curve. The adopted distance notion for two arbitrary curves C_1 and C_2 is a statistical description (in particular the mean) of the signal obtained by taking the norm of the vectors leaving orthogonally from C_1 and meeting C_2 (this will be referred to as the *orthogonal distance*). This analysis proves that a good compromise between the minimisation of the number of parameters and the non-injectivity degree is represented by 20 two-dimensional control points (hence, 40 parameters), with a non-injectivity degree estimated in less then 1 cm.

However, 40 parameters are still numerous if compared with the number of actuators. In order to reduce the number of dimensions for the shape space without affecting the non-injectivity degree, a technique is developed to demote the two-dimensional boundary curve to a one-dimensional signal with no loss of information, aiming at almost reduce by half the number of parameters while maintaining the accuracy of the description. This goal can be reached through two

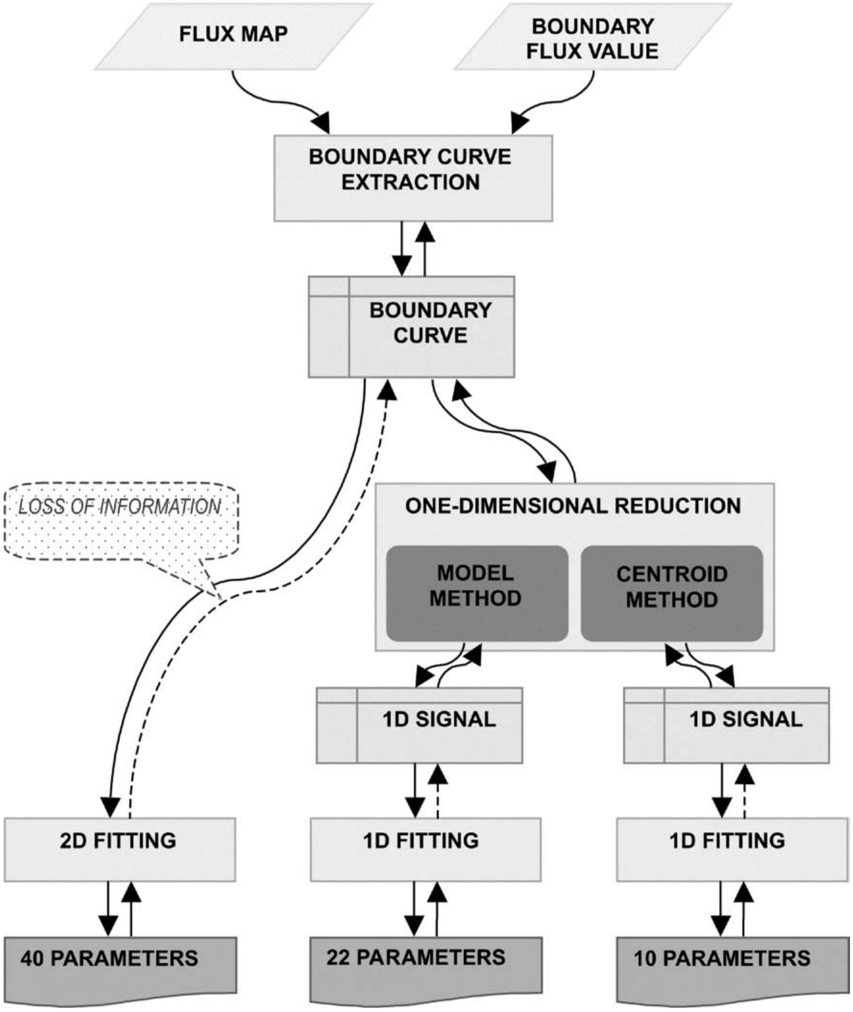

Fig. 1. Flowchart of the boundary reconstruction procedure: from the flux map to the shape parameters. The dashed arrows indicate the non-injectivity of the operation.

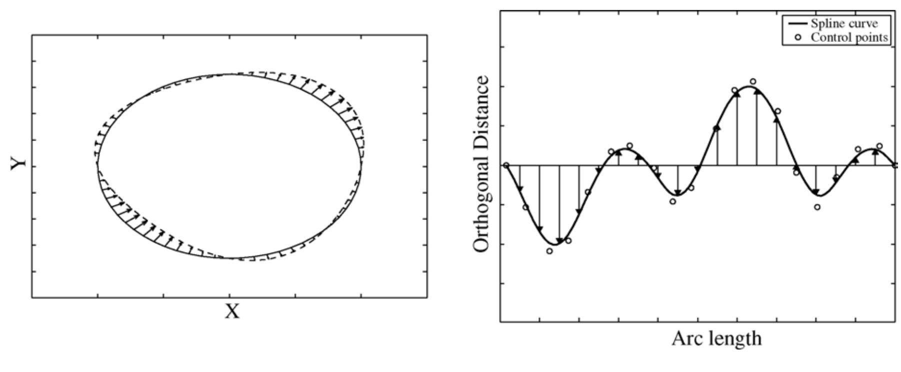

Fig. 2. Shape deformation and orthogonal distance notion (left) and mono-dimensional spline reduction (right).

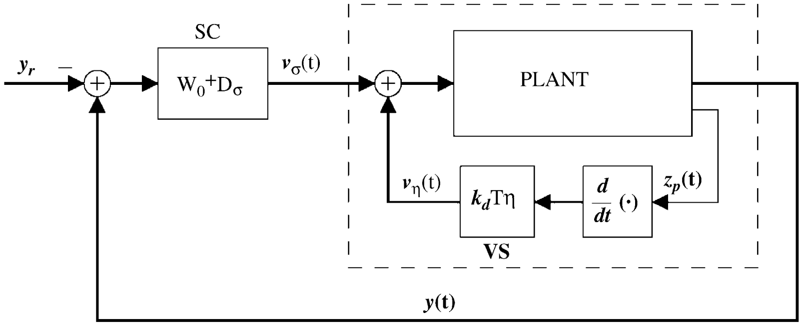

Fig. 3. Schematic diagram of the controlled system. The shape controller (SC) and the vertical stabilisation (VS) are shown as separate blocks.

strategies, called the centroid-method and the model-method. The first one is based on the boundary curve invariant property of being expressed through a particular polar representation on the poloidal plane, where the angular co-ordinate is monotonically increasing: more in detail, it is possible to choose a point inside the curve (centroid) and measure its distance from the boundary points. The model-method takes a model shape as a reference and the one-dimensional signal is obtained by evaluating the orthogonal distance from the model to the actual shape to be parameterised (Fig. 2). As it depends on the model, this

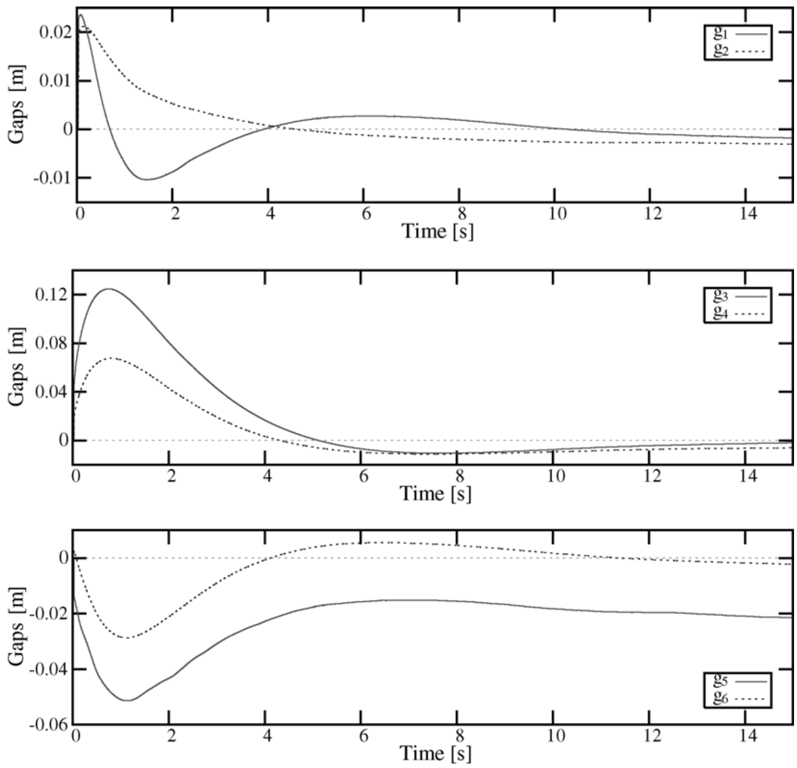

Fig. 4. Gap error behaviour (g_1–g_6) using the gap controller.

second method cannot parameterise both the single null and the limiter shape, conversely to the centroid-method which can be applied to both configurations, and needs a higher number of parameters, when the difference between the model and the actual shape increases.

An analysis of the shape space similar to the study of the quasi-isomorphism for the two-dimensional approach is applied to the one-dimensional representations. The results show that it is possible to obtain for the centroid-method a set of 22 one-dimensional parameters with the same (or less) non-injectivity degree as in the original case. On the other hand, while being less general, the model approach presents extremely good performances, allowing a reduction in the number of parameters to 10 and presenting a more linear behaviour with the variations in the shaping currents. Finally, the possibility of testing the integral technique using almost the same controller as the one adopted for the gap technique suggests the use of the model-method.

3. The B-spline controller: design and preliminary results

The plasma-plant system to be controlled is shown in the schematic of Fig. 3 and can be thought as a dynamic system where the input variables are the actuator voltages **v** and the output quantities are the centroid vertical position of the plasma current z_p, the plasma current itself (I_p) and a generic parameterisation of the boundary shape (these two grouped together and indicated by **y**).

The controller design then proceeds by employing a linearised model of the system around an equilibrium point. This can be cast into a state space representation, where the state **x** of the system comprises the currents

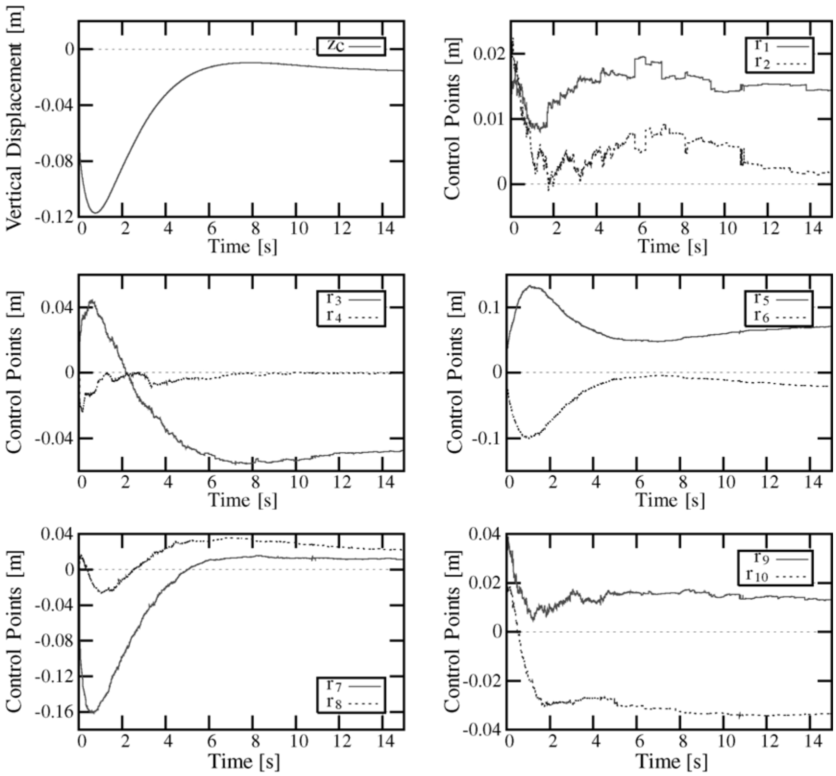

Fig. 5. Control point locations (r_1–r_{10}) and centroid vertical displacement (z_c) using the gap controller.

flowing in the active circuits and those induced in the passive structures. It follows:

$$\begin{bmatrix} \mathbf{L}_{11}^* & \mathbf{L}_{12}^* \\ \mathbf{L}_{21}^* & \mathbf{L}_{22}^* \end{bmatrix} \dot{\mathbf{x}} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \mathbf{v}$$

$$\begin{bmatrix} z_{\mathrm{p}} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_{z_{\mathrm{p}}1} & \mathbf{C}_{z_{\mathrm{p}}2} \\ \mathbf{C}_{\mathbf{y}1} & \mathbf{C}_{\mathbf{y}2} \end{bmatrix} \mathbf{x}$$

where the modified inductance matrices $\mathbf{L}_{ij}^*$ take into account also the presence of the plasma.

Moreover, in the design of the controller, a decoupling scheme is considered, to address the vertical instability issue on a faster timescale, and allow in this way the current and shape controller to operate over an already stabilised plasma.

This strategy, already successfully adopted in the framework of the ITER project (see Portone [6]), has its objective in the derivation of the controller acting on the shape, represented by a set of gaps, in the case of the gap control technique, or summarised by the spline control points, if the integral control technique is exploited.

With some calculations, the transfer function between the shape controller voltages $\mathbf{v}_{\boldsymbol{\sigma}}$ and the output quantities $\mathbf{y}$ can be approximated in the low frequency domain by:

$$\mathbf{W}_{\mathbf{v}_\sigma,\mathbf{y}}(s) \approx \frac{1}{s}\mathbf{W}_{\mathbf{0}},$$

and a control law is assumed as:

$$\mathbf{v}_{\boldsymbol{\sigma}} = W_{\mathbf{0}}^{+}\mathbf{D}_{\sigma}(\mathbf{y} - \mathbf{y}_{\mathbf{ref}}),$$

where $W_{\mathbf{0}}^{+}$ indicates the pseudo-inverse of $\mathbf{W}_{\mathbf{0}}$, $\mathbf{D}_{\boldsymbol{\sigma}}$ the diagonal matrix and $\mathbf{y}_{\mathbf{ref}}$ the vector of the reference values. For the details on the derivation of the controller, we refer the reader to Portone [6] and references within.

In this context, instead, it is important to stress the fact that to assess the validity of the novel boundary representation and moreover the feasibility of an integral approach to the control of the plasma shape, the

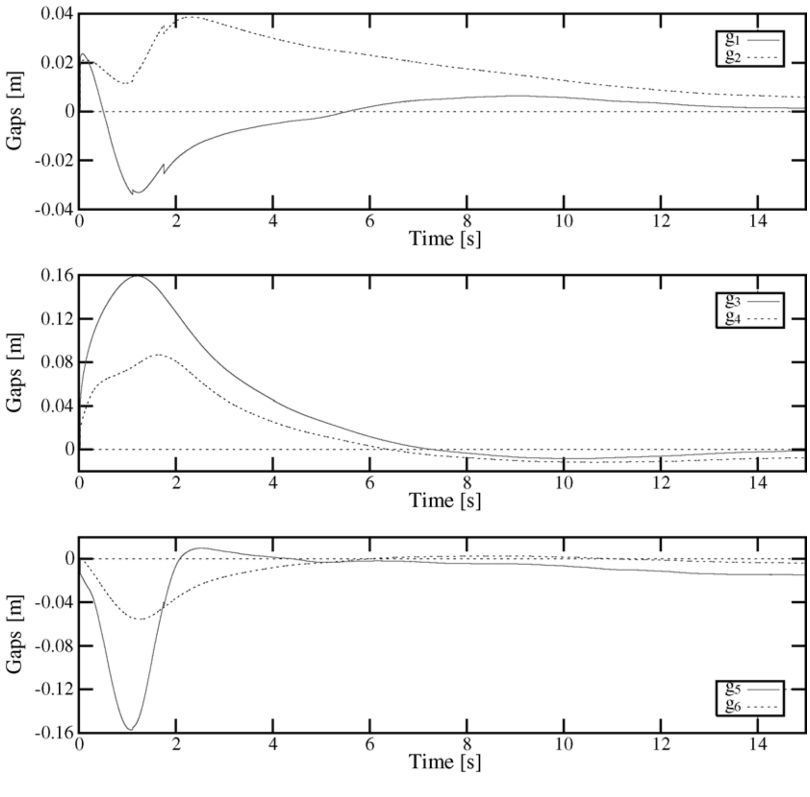

Fig. 6. Gap error behaviour (g_1–g_6) using the spline controller.

control methodology developed for an established gap description has been applied to the spline boundary description with no adaptation or optimisation whatsoever. This is possible because it is not relevant for this study whether the control variables are the control points of a one-dimensional spline representation or the gap distances. In this way, the controller performances are tested beyond the control accuracy and involve robustness issues and flexibility.

In fact, the comparison between the gap controller and the spline controller, the latter only at a germinal stage, is carried out with reference to the ITER design, after having implemented a discrete version of both controllers in the MAXFEA code (Barabaschi [7]) and regards the control of the plasma shape during a minor disruption event. The validation consists in monitoring the control point displacements during the gap controller action and, vice versa, in verifying the regulation of the gap errors while the spline controller is operating.

The results of these non linear analyses show that the gap controller is not efficient in its *side effective* action on the spline control points **r**, and while the gap errors **g** converge towards zero after the application of the disturbance, as reported in Fig. 4, some of the control points remain well distant from their equilibrium values (Fig. 5). On the other hand, when the controller acts directly on the B-spline parameters (equal in number to the available actuators), the concurrent zeroing of the error on gaps is achieved (see Figs. 6 and 7).

This behaviour suggests that the spline representation of the boundary provides a better characterisation of the shape properties, and allows at the same time to reach a more refined accuracy in the control action. In other words, it can be said that as the integral description includes the local description of the parts, a similar relation stands from the controllability point of view, and the spline controller *contains* – in some sense – the gap controller.

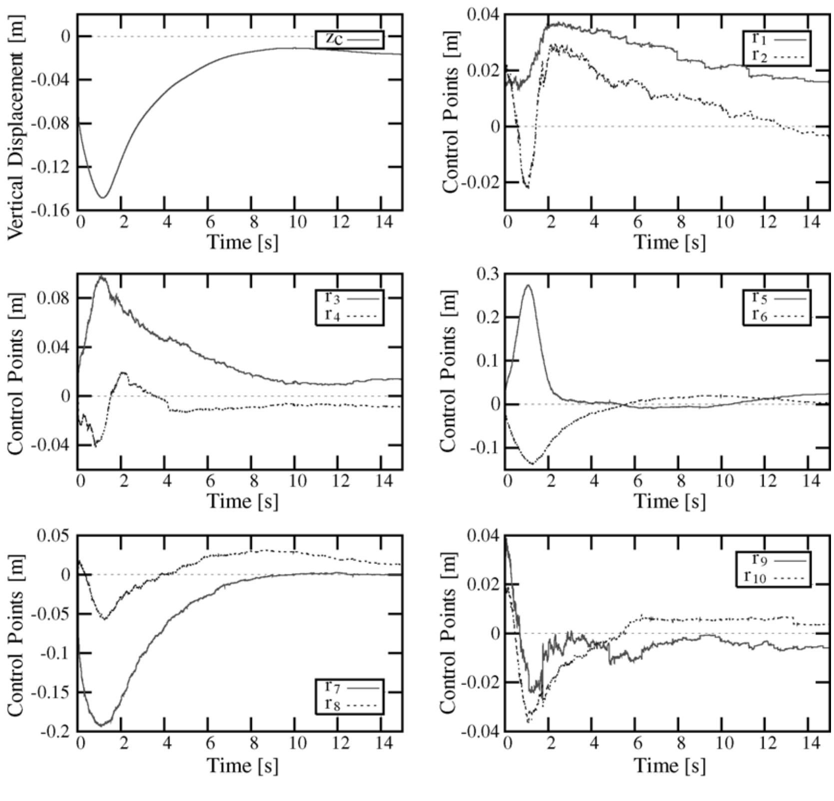

Fig. 7. Control point locations (r_1–r_{10}) and centroid vertical displacement (z_c) using the spline controller.

It is fair to stress once again the overall qualitative value of these comparison and the fact that since the spline controller is not optimised, no attention is paid to the power efficiency or saturations in the actuators either.

4. Conclusion

The technique presented in this article is a first attempt of controlling the boundary of the plasma by means of an integral representation. At the same time, this description provides regularisation on the diagnostic data and enhanced noise rejection is achieved.

On a different ground, the representation provided by the spline approach yields a new definition of integral parameters such as elongation and triangularity, basing the calculations on an area metric instead of exploiting linear measurements (Beghi and Cenedese [8]).

To sum up, this first assessment proves the validity of the approach in terms of control accuracy, and shows that a system designed according to the integral representation contains the features of that obtained from a traditional gap methodology. Although quite preliminary, an interesting result is obtained by monitoring also the integral parameters: once again, the spline controller is capable of make both the elongation and the triangularity converge to the initial equilibrium point better than the gap controller.

More work needs to be done both in evaluating the performance of the new controller in terms of its use of the actuators, and in exploring different design technique so as to optimise the spline controller and improve its efficiency.

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